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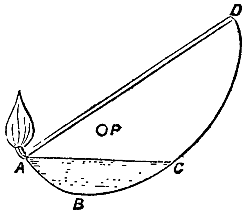
just spilling over in the position of equilibrium, whatever the amount of liquid it contains (1) when it rests on a horizontal plane, (2) when it is suspended about a horizontal axis." This reminds one of problem number 2353 proposed by De Morgan in *The Educational Times* about fifty-five years ago: "The late Dr. Milner, President of Queen's College, Cambridge, constructed a lamp, which General Perronet Thompson remembers to have seen. It is a thin cylindrical bowl, revolving about an axis at P, and the curve $ABCD$ is such that, whatever quantity of oil ABC may be in the bowl, the position of equilibrium is such that the oil just wets the wick at A. What is the curve $ABCD$?" (Cf. *Mathematical Questions with their Solutions from the Educational Times*, volume 7, 1867, p. xvi. A few years later De Morgan referred to the problem in his *A Budget of Paradoxes*, London, 1872, p. 149; second edition by D. E. Smith, 1915, vol. 1, p. 252.) A solution by D. Biddle was published in *Mathematical Questions . . .*, volume 49, 1888, pp. 54–55. He found that a very near approach to the curve required was $r = \cos^{1/2}\theta$. This is one of a family of curves $r = a\cos^n m\theta$, arising in applications of descriptive geometry (cf. Gabriel Marie, *Exercices de Géométrie Descriptive,* 4e éd. Tours, 1909, pp. 835–842; indeed the special case $r = a\cos^{1/2}\theta$ is discussed on page 841).



The problem of the curve for Milner's lamp was considered by Tait, who refers to its formulation in De Morgan's *Budget,* in a paper read before the Edinburgh Mathematical Society in 1887.[1] He quoted De Morgan's statement that the lamp was "a hollow-semi-cylinder, but not with a circular curve," and arrived at a "direct contradiction" of this statement. As a question in connection with the differential equation caused trouble he applied to Cayley who in reply showed,[2] that starting with Tait's differential equation, the solution found was correct. Without any reference to Tait, Biddle discussed the circular form and the consequent lack of "bias to cause rotation." Arc.

PROBLEMS—SOLUTIONS

2799 [1920, 31]. Proposed by H. C. BRADLEY, Massachusetts Institute of Technology.

A newspaper recently gave this problem: Cut a regular six-pointed star into the fewest number of pieces which will fit together and make a square. The newspaper gave a solution in seven pieces. First cut off two opposite points of the star. Divide each into two parts, and fit to the remaining portion of the star so as to make a rectangle. Find the mean proportional between the length and breadth of this rectangle (construction not shown); this is the side of the required square. Using this dimension on the two long sides of the rectangle, divide the latter into three pieces, which make the square. Total seven pieces.

How may the square be formed with not more than five pieces?

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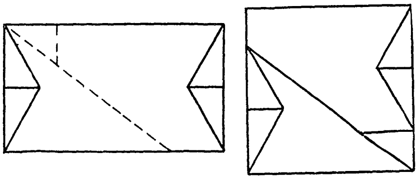
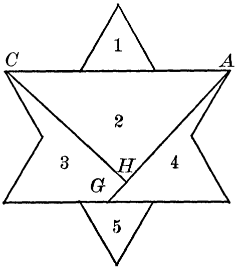
[1] P. G. Tait, "Note on Milner's lamp," *Proceedings of the Edinburgh Mathematical Society,* vol. 5, 1887, pp. 97–98; *Scientific Papers by Peter Guthrie Tait,* vol. 1, 1900, pp. 215–216.

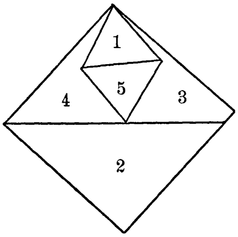
[2] A. Cayley, "On a differential equation and the construction of Milner's lamp," *Proceedings of the Edinburgh Mathematical Society,* vol. 5, 1887, pp. 99–101; *Collected Mathematical Papers of Arthur Cayley,* vol. 13, 1897, pp. 3–5.

SOLUTION BY E. B. ESCOTT, Chicago, Illinois.

Find the side of the equivalent square. Draw AG from one point of star equal to the side of the square. Draw CH perpendicular to AG. Then cut and arrange the pieces as in the figure.







The proposer gave a similar solution and the accompanying figures to illustrate the newspaper solution and his own.—EDITORS.

2819 [1920, 134]. Proposed by B. F. FINKEL, Drury College.

Find the equation of the envelope of the system of circles inscribed in a triangle with a given base and a given vertical angle.

I. SOLUTION AND REMARKS BY H. S. UHLER, Yale University.

Let the rectangular coördinates of the ends of the base of the triangle be $(b, 0)$ and $(-b, 0)$, let 2ϕ be the constant vertical angle, and let γ denote the angle which the bisector of this angle makes with the positive direction of the x-axis.

Since the slope-angles of the bisectors of the base angles at $(b, 0)$ and $(-b, 0)$ are respectively $\frac{1}{2}(\pi + \gamma + \phi)$ and $\frac{1}{2}(\gamma - \phi)$, the equations of these lines may be written

$$y = -(x - b) \cot \tfrac{1}{2}(\gamma + \phi),$$
$$y = (x + b) \tan \tfrac{1}{2}(\gamma - \phi).$$

Solving for x and y, the coördinates of the center of the inscribed circles are found to be

$$\left.\begin{aligned} x_c &= b \cos \gamma/\cos \phi, \\ y_c &= b\,(\sin \gamma - \sin \phi)/\cos \phi. \end{aligned}\right\} \tag{1}$$

Since the radius of the inscribed circle equals y_c, the equation of this circle is

$$x^2 + y^2 - 2x_cx - 2y_cy + x_c^2 = 0. \tag{2}$$

Differentiating equation (2) with respect to γ, and substituting the values of $dx_c/d\gamma$ and $dy_c/d\gamma$ as obtained from equations (1), we obtain

$$\cos \phi \sin \gamma \cdot x - \cos \phi \cos \gamma \cdot y - b \sin \gamma \cos \gamma = 0. \tag{3}$$

Solving the equations (2) and (3) for x and y, with due regard to equations (1), the parametric equations of the envelope are found to be

$$x = x_c, \qquad y = 0. \tag{4}$$

$$\left.\begin{aligned} x &= b[2(\sin \gamma - \sin \phi) \sin \gamma + 1] \cos \gamma/\cos \phi, \\ y &= 2b(\sin \gamma - \sin \phi) \sin^2 \gamma/\cos \phi. \end{aligned}\right\} \tag{5}$$

Equations (4) signify the base of the triangle. This branch of the locus is obviously generated by the lowest point of the inscribed circle as it rolls along the base of the triangle.

The rectangular equation of the other branch may be obtained as follows. Write equation (3) as

$$\cos \gamma = x \cos \phi \sin \gamma/(b \sin \gamma + y \cos \phi)$$